sionally, when the signal weakens gradually but persists for a considerable time, it is possible to increase the sensitivity to the point where the recording relay is chattering continuously. If the instrument contacts are constant in length and regular in position, they may then be found among the random extraneous signals more or less readily, thereby increasing the duration of the flight considerably.

Conclusions and recommendations.—Considering maximum life and least weight, the most favorable combination of batteries for the present model of the Harvard meteorograph is believed to be a filament battery consisting of three 1-inch diameter flashlight cells in series and a plate battery consisting of one small 45-volt unit made of Burgess size V cells. Larger batteries or better heat insulation may become necessary when balloons reaching to higher altitudes are used, or when winds aloft have higher than normal velocities.

It is believed that the additional weight of a small amount of insulating material applied to the batteries would be more than compensated for by reduced weight of batteries required, or by much longer life of the present batteries.

Batteries should, in general, be used within a month of receipt from the factory. In no case should batteries of small special sizes be accepted from retailers or other indirect sources. Filament batteries can best be of standard flashlight cell sizes, obtainable fresh from large stores near the point of use, and used within a few days of purchase. Recent types of batteries with layer built cell construction were used during some of these soundings, and were found to lose voltage very rapidly within 2 weeks after their delivery with the instruments. Experience in these tests indicates that about 100 grams per 45-volt unit is the minimum safe weight of a dry battery as now constructed. Tests on batteries which failed on the shelf showed that their failure was probably due to a failure of a single cell, which made the whole battery useless. Improvement must presumably be in the direction of increased uniformity of chemical mixture and assembly technique.

The new 1.5 volt radio tubes are not considered satisfactory for immediate substitution in place of the present 2-volt tubes, due to their reduced power output and shorter period of operation before cut-off, but should be further investigated for possible use in a new design of the radiometeorograph.

FORMATION OF POLAR ANTICYCLONES

By H. WEXLER

[Weather Bureau, Washington, D. C., April 1937]

INTRODUCTION

Meteorologists have known for a long time that when air is cooled from below over a certain area for an extended length of time, an anticyclone forms. The explanation generally given is the following: Cooling of the air, when confined to a restricted area, lowers the isobaric surfaces and so causes a compensating inflow of air from adjacent regions which raises the surface pressure. However, the mechanism governing the compensating inflow and its distribution with height has not been studied carefully. This problem is the subject of the present paper.

Since the cooling over the area is accompanied by a lowering of the isobaric surfaces, that is, by a deepening of a cyclone situated at some level aloft, it seems natural to apply to the problem the Brunt-Douglas [1] theory of the isallobaric velocity component: When a pressure distribution changes with time, the actual wind is composed of two components, viz, the gradient wind, which prevails during stationary, nonfrictional conditions; and an isallobaric component, which blows into the central region of lowering pressure (isallobaric LOW), and is very nearly proportional to the isallobaric gradient. It will be shown that the transport of air across the isobars by the isallobaric component is sufficient to account for the growth of polar anticyclones that is actually observed on weather maps.

Before the isallobaric velocity component can be applied, it is necessary to adopt a cooling model for the atmosphere. In a recent paper by the author,[2] a cooling model was presented for a calm, cloudless, sunless atmosphere possessing, initially, a steep lapse rate and underlain by an unlimited snow surface. From the radiation exchange between the snow surface and the atmosphere, it was possible to determine a relation between the temperature of the snow surface and the maximum free-air temperature. For the snow surface temperature to fall below the value given by this relation, the maximum free-air temperature must decrease; and the cooling process will be one whereby the atmosphere loses energy to space mostly through the spectral band in the black-body radiation from the snow surface to which water vapor is transparent. As the cooling continues, the steep lapse rate decreases until, finally, the atmosphere becomes practically isothermal from above the shallow surface layer of cold air to a height dependent on the initial lapse rate (see fig. 1).

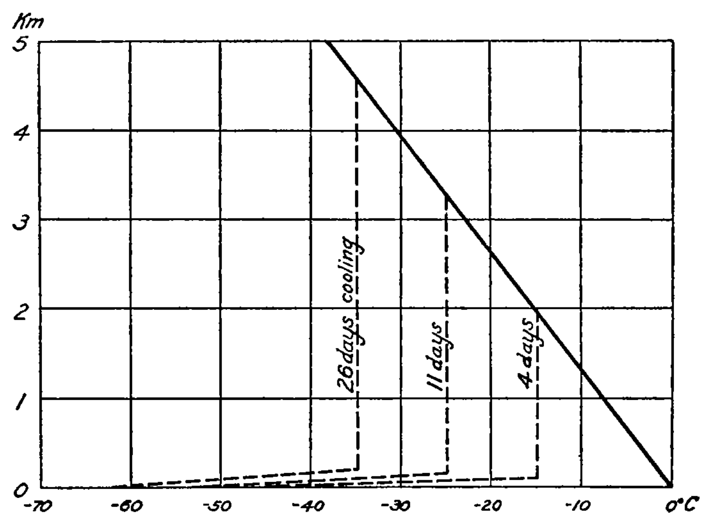

FIGURE 1.—Transformation of polar maritime air into polar continental air.

The cooled air mass is what may properly be called polar continental air, and is found over extensive land and frozen maritime areas in high latitudes during winter. The air above the cooled layer is still characterized by a steep lapse rate, and it also cools by means of radiation directly

[1] Mem. Roy. Met. Soc., Vol. III, No. 22, September 1928.
[2] Mo. WEA. REV., Vol. 64, p. 122, April 1936.

to space; however, it can be shown [2,3] that the cooling of this air is much less than that of the air closer to the surface. In this paper the loss of energy directly to space from the air above the polar continental air will be neglected at first, and later taken into account in a qualitative manner.

Let us now consider the following problem. Assume that the region surrounding the North Pole is a uniform snow field at sea-level and is covered by an extensive air mass of horizontal homogeneity but possessing a vertical temperature gradient of 7.6° C. per kilometer and a surface temperature of 0° C. At the autumnal equinox, the sun is just leaving the Pole; and at the end of 37 days, the region of darkness will have extended to about 1,500 kilometers from the Pole (at about 76° latitude). If during this period 70 percent clear sky has prevailed, then at the Pole the equivalent of 26 days of cooling will have occurred, and at 1,500 kilometers from the Pole there will have been 0 days of cooling. As can be seen from figure 7 of the previous paper,[2] in which it is assumed no heat is transported upward through the snow cover, this cooling will result in a surface temperature of −62° C., and an isothermal layer temperature of −35° C., at the Pole. With the lapse rate we have assumed, the height of the isothermal layer (depth of polar continental air) will be 4.6 kilometers. At 1,500 kilometers from the Pole the depth of the polar continental air will be 0, and, as in figure 2, a wedge of

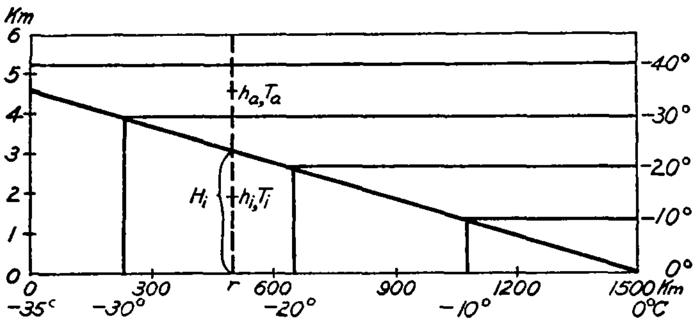

FIGURE 2.—Structure of the wedge of polar continental air.

polar continental air will occupy a circular region of 1,500 kilometers radius centered about the Pole. The isotherms are also shown, but are not drawn to show the existence of the shallow layer of cold air next to the ground. In the following developments, this layer will be neglected on the assumption that it is of negligible thickness.

The analysis to be given here is not restricted to the region surrounding the geographic pole, but can be applied with slight modifications to a region in the neighborhood of any "cold pole", arbitrarily located. In the northern hemisphere in winter, such cold poles are found in northern Asia and in Canada and Alaska. To apply the following methods to an elevated cold pole, however, such as the Greenland ice-cap or the Antarctic Plateau, would be more difficult, since account would have to be taken of the drainage of the cold air to sea-level.

The rate of cooling of the isothermal layer under the conditions assumed above is shown in figure 3 as a solid curve, and is represented analytically by a complicated function of time. However, in order to avoid undue complexities in such an idealized model as adopted here, a linear rate of cooling is assumed, such that in 26 days of cooling, the isothermal layer cools from 0° C. to −35° C; then, if t is the number of seconds of cooling, and T_0 the temperature in °A of the polar continental air, we have

[2] Mo. WEA. REV., Vol. 64, p. 122, April 1936.
[3] Mo. WEA. REV., Vol. 65, p. 102, March 1937.

(1) $$T_0=273-\beta t, \text{ where } \beta=156\cdot10^{-7}$$

The curve plotted from (1) is shown in figure 3 as a dashed line.

Now up to about latitude 72° the recession of the sun southward is very nearly linear with time, and so at any given time the latitudinal temperature distribution will be linear in r, the distance from the Pole. The temperature of the polar continental air at time t, and distance r (in cm.), is given by

(2) $$T_i=273-\beta t+\alpha r, \text{ where } \alpha=2.33\cdot10^{-7},\ \beta=156\cdot10^{-7};$$

t and r are independent variables, but $0\leq r\leq\xi$, $\xi=67\,t$.

COMPUTATION FOR ANTICYCLOGENESIS CAUSED BY CONVERGENCE IN THE POLAR CONTINENTAL AIR

Let us first compute the magnitude of the anticyclogenesis that results from cooling and compensating inflow in the polar continental air alone.

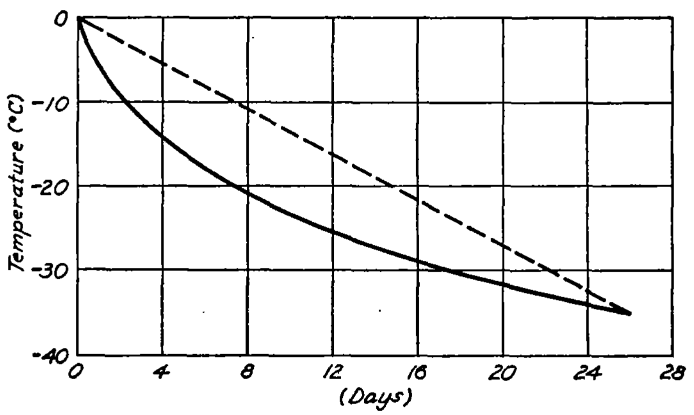

FIGURE 3.—Actual and assumed cooling curves for the isothermal layer (polar continental air).

At a temperature T_i, and at a height h_i above sea level, the pressure in a vertically isothermal atmosphere is given by

(3) $$p_{h_i}=p_0e^{-\frac{mg}{RT_i}h_i}=p_0e^{-\frac{h_i}{H}},$$

where p_0 is the pressure at the surface, assumed to be constant initially; $\frac{R}{m}$ is the gas constant for dry air; g is acceleration due to gravity; and H is the height of the homogeneous atmosphere of surface temperature T_i.

Differentiating with respect to r, and holding p_0 constant, the pressure gradient at height h_i is found to be,

(4) $$\frac{\partial p_{h_i}}{\partial r}=\frac{mgh_i\alpha}{R}\frac{p_{h_i}}{T_i^2}.$$

However, the assumption that p_0 is constant at some stage of the anticyclogenesis is erroneous, since it is the change in p_0 that is desired. Simultaneously with the formation of the cyclone aloft (polar cyclone), the surface anticyclone forms and, as cooling continues, increases in height. If p_0 is held constant, then we assume that the polar cyclone exists at all levels from just above the surface; and we will obtain too great an accumulation of air from inflow, because actually the building-up of the surface anticyclone diminishes the inward transport of air at all heights and really causes an outflow of air in the surface layers. On the other hand, if the variation in p_0 is included in the computations, then a differential equation results which it seems most practicable to solve by a method of successive approximation, as will be shown later: In the first approximation, p_0 is considered constant at 1,000 mb., and the increase of surface pressure found will be too large. Later, when the second approximation is discussed, it will be shown that the maximum difference between the two solutions is less than 24 percent.

Another restriction on the first simple approximation is the assumption of linear flow only. The case of circular flow will be taken up later in this paper.

The geostrophic wind G_i is, from (4),

$$G_i=\frac{1}{\rho_{h_i} l}\frac{\partial p_{h_i}}{\partial r}=\frac{gh_i\alpha}{l}\frac{1}{T_i}, \tag{5}$$

where $l=2\Omega\sin\phi$, assumed constant at 1.41×10^{-4}; Ω is the angular velocity of the earth's rotation, ϕ is latitude, and ρ_{h_i} is density at height h_i.

Now the isallobaric velocity component into the deepening Polar Cyclone at height h_i, time t, and distance from Pole r, is found by differentiating (5) with respect to time:

$$\frac{1}{l}\frac{\partial G_i}{\partial t}=\frac{gh_i\alpha\beta}{l^2}\frac{1}{T_i^2}. \tag{6}$$

The transport of mass across the isobars is

$$\frac{\rho_{h_i}}{l}\frac{\partial G_i}{\partial t}=\frac{g\alpha\beta\rho_o}{l^2T_i^2}h_i\,e^{-h_i/H}\text{ grm/cm}^2\text{/sec}, \tag{7}$$

where ρ_o is the density at the surface.

The transport reaches a maximum at $h_i=H$; hence, for an atmosphere with zero vertical temperature gradient, but positive horizontal temperature gradient, the mass transport into the central region of cooling attains a maximum at the height of the homogeneous atmosphere corresponding to surface temperature T_i. When $T_i=259°$, $H=7.6$ km.

The total inward transport between the surface and height h_i is

$$I_1=\frac{g\alpha\beta\rho_o}{l^2T_i^2}\int_0^{h_i}h_i\,e^{-h_i/H}\,dh_i \tag{8}$$

$$=\frac{p_o\alpha\beta H}{l^2T_i^2}\left[1-\left(1+\frac{h_i}{H}\right)e^{-h_i/H}\right]\text{ grm/cm}^2\text{/sec}\cdot$$

If we now consider the flow of air into a cylinder of radius 900 kilometers, whose height, 1.8 kilometers, is that of the polar continental air at that distance from the center, the average increase of surface pressure in mb. for 24 hours is 0.093, which is too small to account for polar anticyclogenesis as actually observed. If, however, the cylinder extends to the top of the atmosphere ($h=\infty$), the average increase in pressure is 3.9 mb./day.

COMPUTATION FOR ANTICYCLOGENESIS CAUSED BY CONVERGENCE IN THE SUPERIOR AIR

If the height of the polar continental air is designated by H_i, then H_i is given by

$$T_i=273-7.6\times10^{-5}H_i. \tag{9}$$

If T_a is the temperature of the superior air at height h_a, then

$$T_a=273-7.6\times10^{-5}h_a.$$

Now the pressure, p_{h_a}, at h_a is given by

$$p_{h_a}=p_oe^{-\frac{mg}{RT_i}H_i}\left(\frac{T_a}{T_i}\right)^{4.5}$$

$$=p_oe^{-4.5\frac{273-T_i}{T_i}}\left(\frac{T_a}{T_i}\right)^{4.5},$$

where T_i alone depends on r.

Then

$$\frac{\partial p_{h_a}}{\partial r}=4.5\alpha p_{h_a}\frac{273-T_i}{T_i^2},$$

and

$$G_a=\frac{1}{l\rho_{h_a}}\frac{\partial p_{h_a}}{\partial r}=\frac{gH_i\alpha}{l}\frac{T_a}{T_i^2}. \tag{10}$$

The isallobaric velocity is

$$\frac{1}{l}\frac{\partial G_a}{\partial t}=\frac{g\alpha\beta}{l^2}\frac{10^5}{7.6}\left(\frac{2\cdot273-T_i}{T_i^3}\right)T_a; \tag{11}$$

and the transport of air across the isobars is

$$I_a=\frac{\rho_{h_a}}{l}\frac{\partial G_a}{\partial t}=\frac{10^5}{7.6}\frac{g\alpha\beta}{l^2}\left(\frac{2\cdot273-T_i}{T_i^3}\right)T_a\rho_oe^{-4.5\frac{273-T_i}{T_i}}\left(\frac{T_a}{T_i}\right)^{3.5}; \tag{12}$$

the total transport through a strip of 1 cm. width reaching from the top of the polar continental air to h_a is

$$I_2=\int_{H_i}^{h_a}I_a dh_a=-\frac{10^5}{7.6}\int_{T_i}^{T_a}I_a dT_a \tag{13}$$

$$=\frac{10^5\cdot4.5\alpha\beta}{7.6\cdot5.5\,l^2}p_o(2\cdot273-T_i)\frac{e^{-4.5\frac{273-T_i}{T_i}}}{T_i^2}\left[1-(T_a/T_i)^{5.5}\right]$$

For a circular region of 900 km. radius, the average increase in pressure resulting from convergence in the superior air from 1.8 km. to the top of the atmosphere (36 km.) is 12.40 mb/day, and from 1.8 km. to 8 km. (top of the troposphere) is 8.26 mb/day, and the total average pressure increase is 8.26+0.09=8.35 mb/day, which is a more reasonable value. Almost the entire pressure increase observed is due to the convergence of the air above the polar continental air. This is by no means explained by the greater thickness of this layer; in the following section it will be shown that a much more important factor enters, namely, a many-fold increase in isallobaric velocity at the front, in going from the polar continental air to the air above.

THE ISALLOBARIC VELOCITY DISCONTINUITY

From equations (5) and (10), we see that at the front, between the polar continental air and the air above, where $h_i=H_i$ and $T_a=T_i$, there exists continuity in geostrophic wind velocity. However, as seen from equations (6) and (11), there exists a discontinuity in isallobaric velocity at the front:

$$\frac{1}{l}\frac{\partial G_i}{\partial t}=\frac{g\alpha\beta H_i}{l^2\,T_i^2}=\frac{g\alpha\beta}{l^2}\frac{10^5}{7.6}\frac{273-T_i}{T_i^2} \tag{6}$$

$$\frac{1}{l}\frac{\partial G_a}{\partial t}=\frac{g\alpha\beta}{l^2}\frac{10^5}{7.6}\frac{2\cdot273-T_i}{T_i^2}; \tag{11}$$

the ratio of the two velocities is $\frac{2\cdot273-T_i}{273-T_i}$ which is equal to 9 at the center of the wedge where the front is highest, and is equal to 20 at 900 kilometers from the center where the front is lower.

The explanation of this discontinuity involves a closer examination of the Brunt-Douglas derivation of the isallobaric velocity:

Consider a 2-dimensional system of coordinates, having its X-axis pointing to the east and its Y-axis pointing to the north. The equations of motion for frictionless flow on a restricted portion of the earth's surface are

$$(a)\qquad \frac{du}{dt}=lv-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$(b)\qquad \frac{dv}{dt}=-lu-\frac{1}{\rho}\frac{\partial p}{\partial y}.$$

Letting $V=u+iv$ $(i=\sqrt{-1})$, multiplying equation (b) by i, and adding,

$$(c)\qquad \frac{d}{dt}(u+iv)=-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}\right)-li(u+iv);$$

solving for $(u+iv)$,

$$(d)\qquad u+iv=\frac{i}{l}\frac{d}{dt}(u+iv)+\frac{i}{\rho l}\left(\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}\right)$$
$$=\frac{i}{l\rho}\left(\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}\right)+\frac{i}{l}\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)(u+iv).$$

Differentiating the above equation,

$$(e)\qquad \frac{\partial(u+iv)}{\partial t}=\frac{i}{l\rho}\left(\frac{\partial \dot p}{\partial x}+i\frac{\partial \dot p}{\partial y}\right)+\frac{i}{l}\left(\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}\right)\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)+\frac{i}{l}\frac{\partial}{\partial t}\frac{d}{dt}(u+iv),$$

where $\dot p=\frac{\partial p}{\partial t}$.

Brunt and Douglas neglect the second term on the right-hand side; but, since in this discussion the density is changing, it is important to retain this term.

From (d),

$$\frac{\partial(u+iv)}{\partial t}=\frac{l}{i}(u+iv)-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}\right)-\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)(u+iv);$$

substituting in equation (e) and solving for $(u+iv)$, we find

$$(f)\qquad V=u+iv=\frac{i}{l\rho}\left(\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}\right)-\frac{1}{l^2\rho}\left(\frac{\partial \dot p}{\partial x}+i\frac{\partial \dot p}{\partial y}\right)$$
$$-\frac{1}{l^2}\left(\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}\right)\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)+\frac{i}{l}\left(u\frac{\partial}{\partial x}+v\frac{\partial}{\partial y}\right)(u+iv)$$
$$-\frac{1}{l^2}\frac{\partial}{\partial t}\frac{d}{dt}(u+iv).$$

Brunt and Douglas maintain that for atmospheric motion the last term of equation (f) is negligible in comparison with the others, and that for flow not confined to stream-lines of too small radius of curvature, the next-to-last term is also negligible. If these terms are neglected, then equation (f) becomes

$$(f)\qquad V=u+iv=G+I_i+I_\rho,$$

where

$$G=\frac{i}{l\rho}\left(\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}\right)=\text{geostrophic velocity},$$

$$I_i=-\frac{1}{l^2\rho}\left(\frac{\partial \dot p}{\partial x}+i\frac{\partial \dot p}{\partial y}\right)=\text{isallobaric velocity},$$

$$I_\rho=-\frac{1}{l^2}\left(\frac{\partial p}{\partial x}+i\frac{\partial p}{\partial y}\right)\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right).$$

In figure 4 these velocities are shown for the polar continental air and the superior air at a point of the front 1.8 km above the ground. The isallobaric velocity in the polar continental air is less than $\frac{1}{10}$ that in the superior air; also, because the density of polar continental air is increasing with time while the reverse is true for the superior air, the quantity I_ρ tends to diminish the isallobaric velocity in the polar continental air, but to increase it in the superior air; hence, the resultant inflow in the lower air is only 0.49 cm/sec., while in the superior air it is 10.05 cm/sec., about 20 times as large.

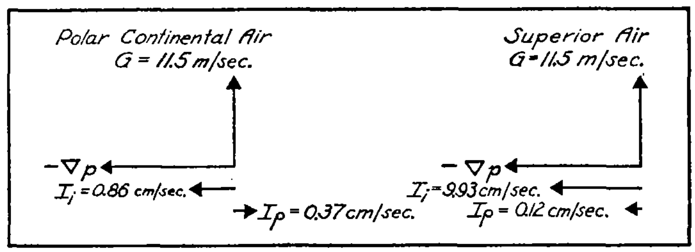

FIGURE 4.—Geostrophic and non-geostrophic velocities at the front (schematic).

The deviation from geostrophic direction in the superior air, caused by the deepening polar cyclone, is about 1:100 and the mass transport of tropospheric air across isobars caused by this slight deviation is sufficient to account for an average increase of pressure of more than 8 mb./day.

It still remains to give a physical explanation of why the isallobaric velocity is 10 times as large in the superior air as in the polar continental air. In order to do this, in figure 5 the pressure tendency profile across the wedge at a height of 1.8 kilometers has been plotted. At the front there appears a discontinuity in the slope of this profile, so that in the superior air it is about 10 times as

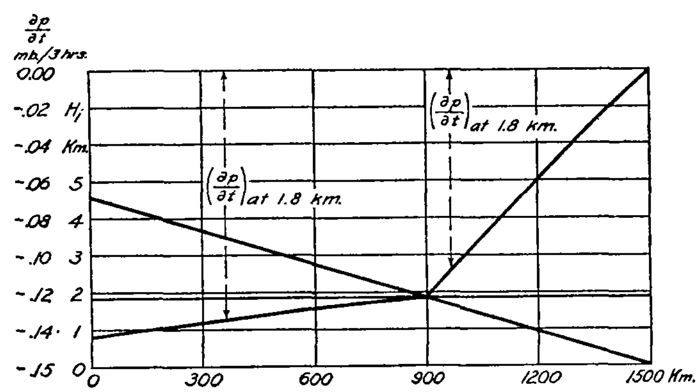

FIGURE 5.—Pressure tendency profile at 1.8 kilometers above the surface.

large as that in the polar continental air. The isallobaric velocity, being proportional to the isallobaric gradient, of course undergoes a similar increase, as already noticed. To explain the discontinuity in isallobaric gradient at the front, imagine that a series of barographs are situated along the wedge at a height of 1.8 kilometers. The barographs in the polar continental air will show almost equal falls, because the sinking of isobaric surfaces is caused mainly by the cooling of the air below 1.8 kilometers.

However, for the barographs situated in the superior air, the only air that is being cooled is the polar continental air which does not reach up to 1.8 kilometers, and so the isobaric surfaces are not sinking as rapidly as in the former case. This becomes more evident as we approach the edge of the polar continental air where the pressure tendency is zero. The discontinuity in the slope of the curve, marking the height of the cooled air which is encountered at 900 kilometers from the Pole, accounts for the discontinuity in the isallobaric gradient, and also for the discontinuity in the isallobaric velocity. To this effect is added that due to the fact that the cooling in the polar continental air increases its density, while the lowering of pressure and the stationary temperature in the superior air decrease its density. These two effects, which combine to give a twentyfold discontinuity in isallobaric velocity at the front, result from the particular cooling model adopted here and illustrated in figures 1 and 2; but the same discontinuity will exist to a less marked degree even if the superior air cools also, provided it does not do so as rapidly as the lower air.

We have previously seen that if the cooling rate is the same at all levels in a vertically isothermal atmosphere, the compensating inflow of air seems too small to account for anticyclogenesis as observed, unless the inflow extends up to the top of the atmosphere. The same would be found true for an atmosphere of steep lapse rate. Hence it appears that in order for polar anticyclogenesis to occur at a rapid rate it is essential that the cooling proceed faster in lower levels than in upper levels.

It now becomes possible to plot the vertical distribution of mass inflow (see fig. 6). From equation (7) it is seen

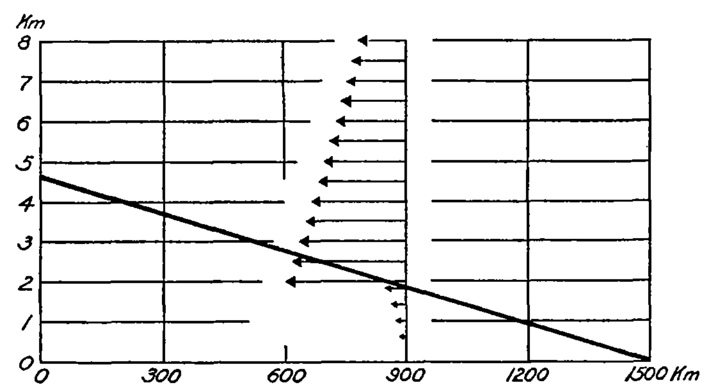

FIGURE 6.—Vertical distribution of mass inflow (arrows shown in the lower air are drawn about 3 times too large).

that the mass transport increases with height until the height of the homogeneous atmosphere is reached (about 7 kilometers). Since the height of the polar continental air is less than this value, the inflow increases with height in the lower air. In the superior air at the front there is about a twenty-fold increase in this inflow, and, as seen from equation (12), the inflow then diminishes with elevation. From equation (13) the ratio of the mass inflow of the superior air, up to various heights, to the total inflow can be found and is shown in table 1.

Since the tropopause is located at about 8 kilometers, the solution found above is valid only up to this height; hence, it can be said that of the total mass inflow occurring between the front (1.8 kilometers) and the tropopause (8 kilometers), about one-half is contained in the 2½ kilometers layer adjacent to the front.

Since anticyclogenesis is occurring at lower levels, in the polar continental air, an outflow really exists in these levels. This decreases with elevation and becomes an inflow at the height corresponding to the line of separation of easterlies from the westerlies.

TABLE 1.—*Ratio of mass inflow of the superior air up to height h_a, and the total inflow*

h_a	T_a	Ratio to total inflow	Ratio to total inflow up to 8 kilometers
1.8 kilometers (at the front)	259	0	0
4	243	.29	.43
6	227	.51	.76
8	212	.67	1.00

COMPUTATION OF THE PRESSURE TENDENCY PROFILE ACROSS AN ANTICYCLONIC WEDGE

If I is the mass of air in grams per second flowing through a strip 1 centimeter wide reaching from the ground to 8 kilometers, then the pressure increase at the ground is given by

$$\frac{\partial p_o}{\partial t}=86400\times 0.98\frac{\partial I}{\partial r}\text{ in mb/day;} \tag{14}$$

where $I=I_1+I_2$, and

$$I_1=\frac{p_o\alpha\beta H}{l^2T_i^2}\left[1-\left(1+\frac{H_i}{H}\right)e^{-H_i/H}\right] \tag{8}$$

$$=\frac{p_o\alpha\beta R/m}{l^2gT_i}\left[1-\left(1+4.5\frac{273-T_i}{T_i}\right)e^{-4.5\frac{273-T_i}{T_i}}\right]$$

$$I_2=\frac{10^5\times 4.5}{7.6\times 5.5}\frac{\alpha\beta}{l^2}p_o(2\times 273-T_i)\frac{e^{-4.5\frac{273-T_i}{T_i}}}{T_i^2}\left[1-\left(\frac{T_a}{T_i}\right)^{5.5}\right] \tag{13}$$

Differentiating (8) and (13) with respect to r, and substituting in (14), the tendency profile across the polar anticyclonic wedge (where the motion is linear) can be found and is shown in figure 7 as the solid curve.

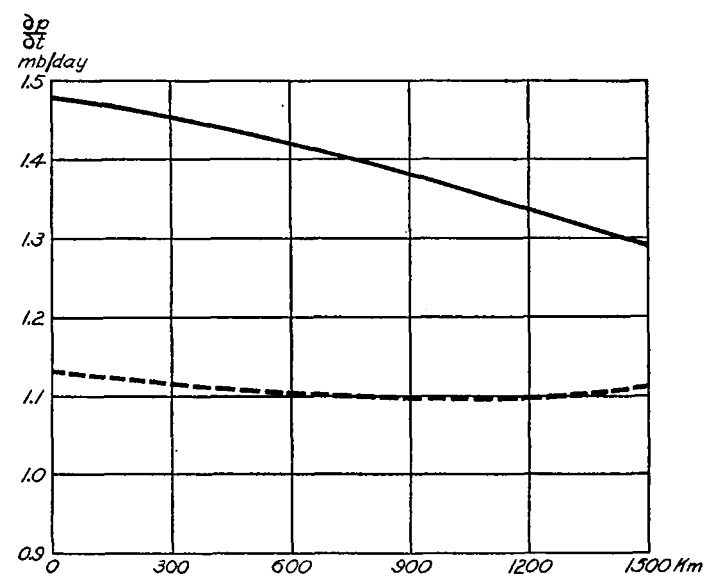

FIGURE 7.—First and second approximations for the pressure tendency profile across the anticyclonic wedge (linear flow).

The pressure changes seem small compared to those observed. However, when a circular anticyclonic area is considered, these turn out to be larger. Before studying this case it is necessary first to compute the isallobaric velocity component for curved motion.

COMPUTATION OF THE PRESSURE TENDENCY PROFILE ACROSS A CIRCULAR ANTICYCLONE

Consider the equations of motion expressed in polar coordinates, (r, θ), where r is the distance of a point from the axis of motion, and θ is the angle of the radius vector, measured counter-clockwise:

$$(a) \qquad \ddot{r}-lr\dot{\theta}-r\dot{\theta}^2=-\frac{1}{\rho}\frac{\partial p}{\partial r},$$

$$(b) \qquad r\ddot{\theta}+2\dot{r}\dot{\theta}+l\dot{r}=-\frac{1}{\rho}\frac{\partial p}{r\partial\theta},$$

where $\dot{r}=\frac{dr}{dt}$, $\dot{\theta}=\frac{d\theta}{dt}$.

Now if the isobars are circular, $\frac{\partial p}{\partial\theta}=0$; therefore,

$$(c) \qquad r\ddot{\theta}+2\dot{r}\dot{\theta}+l\dot{r}=0.$$

Differentiating equation (a) with respect to t, and rearranging terms,

$$(d) \qquad -\dddot{r}+\dot{r}(l\dot{\theta}+\dot{\theta}^2)+r\ddot{\theta}(l+2\dot{\theta})=\frac{d}{dt}\left(\frac{1}{\rho}\frac{\partial p}{\partial r}\right)$$
$$=\frac{\partial}{\partial t}\left(\frac{1}{\rho}\frac{\partial p}{\partial r}\right)+\dot{r}\frac{\partial}{\partial r}\left(\frac{1}{\rho}\frac{\partial p}{\partial r}\right).$$

Assuming $\dddot{r}$ to be negligible in comparison with the remaining terms, eliminating $r\ddot{\theta}$ by means of equation (c), and solving for $\dot{r}$, we find

$$(e) \qquad \dot{r}=\frac{-\frac{1}{l}\frac{\partial G}{\partial t}}{\frac{1}{l}\frac{\partial G}{\partial r}+1+\frac{3\dot{\theta}}{l}+\frac{3\dot{\theta}^2}{l^2}},$$

where $G=\frac{1}{\rho l}\frac{\partial p}{\partial r}$ is the geostrophic wind corresponding to the pressure gradient.

Now for a gradient wind, $\ddot{r}=0$, and so equation (a) becomes $lr\dot{\theta}+r\dot{\theta}^2=\frac{1}{\rho}\frac{\partial p}{\partial r}$, or $\frac{\dot{\theta}}{l}+\frac{\dot{\theta}^2}{l^2}=\frac{1}{rl}G$.

Hence, equation (e) becomes

$$(15) \qquad \dot{r}=\frac{-\frac{1}{l}\frac{\partial G}{\partial t}}{1+\frac{3G}{lr}+\frac{1}{l}\frac{\partial G}{\partial r}}.$$

Now, in general, for cyclonic motion, the denominator will be greater than unity, and so the isallobaric velocity component for cyclonic motion will be less than that for linear motion, which is to be expected, since the centrifugal force tends to act counter to the acceleration caused by the deepening cyclone.

As $r\rightarrow\infty$, the second term in the denominator of equation (15)$\rightarrow 0$, and since the last term is usually quite small, the isallobaric component becomes nearly equal to that for straight-line isobars.

At the center of the cyclone $\dot{r}$ should vanish. This is seen to be true for motion in the polar continental air. From equation (5)

$$\lim_{r\rightarrow 0}\frac{G}{r}=\lim_{r\rightarrow 0}\frac{gh_t\alpha}{l}\frac{1}{r(T_o+\alpha r)}=\infty;$$

therefore

$$\lim_{r\rightarrow 0}\dot{r}=0.$$

Also, for motion in the superior air, $\dot{r}=0$ at $r=0$, as seen from equation (10),

$$\lim_{r\rightarrow 0}\frac{G}{r}=\lim_{r\rightarrow 0}\frac{g\alpha T_a}{l}\frac{H_t}{rT_t^2}=\infty, \text{ for } H_t\neq 0;$$

at $H_t=0$, $\lim_{r\rightarrow 0}\dot{r}$ has no meaning, so that at the initial instant the limiting process does not hold.

In applying equation (15) to the inflow in the polar continental air, it is necessary to have the following expressions:

$$(5) \qquad G=\frac{gh_t\alpha}{l}\frac{1}{T_t},$$

$$(6) \qquad \frac{1}{l}\frac{\partial G}{\partial t}=\frac{gh_t\alpha\beta}{l^2}\frac{1}{T_t^2},$$

$$\frac{1}{l}\frac{\partial G}{\partial r}=-\frac{gh_t\alpha^2}{l^2}\frac{1}{T_t^2},$$

and so

$$\dot{r}=\frac{-\frac{gh_t\alpha\beta}{l^2}\frac{1}{T_t^2}}{1+\frac{3gh_t\alpha}{l^2T_tr}-\frac{gh_t\alpha^2}{l^2}\frac{1}{T_t^2}}.$$

The mass transport to height H_t is

$$(16) \quad I_1=-\int_o^{H_t}\dot{r}\rho dh_t=\frac{\rho_o g\beta}{l^2T_t^2}\int_o^{H_t}\frac{e^{-h_t/H}h_t dh_t}{1+\left(\frac{3}{r}-\frac{\alpha}{T_t}\right)\frac{gh_t}{l^2T_t}}$$
$$=\frac{p_o\beta r}{g(3T_t-\alpha r)}\{1-e^{-x}+\sigma e^{\sigma}[E_i(-\sigma)-E_i(-(x+\sigma))]\},$$

where

$$x=\frac{H_t}{H}=4.5\frac{273-T_t}{T_t}$$

$$\sigma=\frac{l^2T_tr}{\frac{R}{m}\alpha(3T_t-\alpha r)},$$

and $E_i(-\sigma)$ is the exponential-integral function which is tabulated in Jahnke-Emde,[4] p. 83.

The pressure change at the surface, due to convergence in the polar continental air, is given by

$$(17) \qquad \left(\frac{\partial p_o}{\partial t}\right)_1=86400\times 0{\cdot}98\left(\frac{\partial I_1}{\partial r}+\frac{I_1}{r}\right)\text{mb/day}.$$

It is easy to show by means of equation (16) that at $r=0$, $\frac{I_1}{r}$ and also $\frac{\partial I_1}{\partial r}$ are finite. The only term where the question of a limit may arise as $r\rightarrow 0$ is $\sigma E_i(-\sigma)$. However, as shown by Jahnke-Emde,[4] p. 79, for small values of σ, $E_i(-\sigma)\sim ln\gamma\sigma$, $\gamma=1.78$; hence, neglecting signs,

$$\lim_{r\rightarrow 0} ln\gamma\sigma=\lim_{r\rightarrow 0}\frac{ln\gamma\sigma}{\frac{1}{\sigma}}$$
$$=\lim_{r\rightarrow 0}\frac{\frac{1}{\sigma}}{\frac{1}{\sigma^2}}$$
$$=\lim_{r\rightarrow 0}\sigma$$
$$=0$$

[4] Funktionentafeln, B. G. Teubner, Leipzig and Berlin, 1933.

In applying equation (15) to the inflow in the superior air, it is necessary to have the following expressions:

$$(10)\qquad G=\frac{g\alpha T_a H_t}{lT_t^2}=\frac{g\alpha}{l}\frac{10^5}{7.6}\left(\frac{273-T_t}{T_t^2}\right)T_a,$$

$$(11)\qquad \frac{1}{l}\frac{\partial G}{\partial t}=\frac{g\alpha\beta 10^5}{l^2\, 7.6}\left(\frac{2\cdot 273-T_t}{T_t^3}\right)T_a,$$

$$\frac{1}{l}\frac{\partial G}{\partial r}=-\frac{g\alpha^2}{l^2}\frac{10^5}{7.6}\left(\frac{2\cdot 273-T_t}{T_t^3}\right)T_a.$$

Substituting in (15) and integrating the product of $\dot{r}$ and ρ_{h_a} to height h_a, we find

$$(18)\qquad \frac{I_2}{r}=\frac{4.5p_o\beta(2\cdot 273-T_t)e^{-4.5\frac{273-T_t}{T_t}}}{g[3T_t(273-T_t)-\alpha r(2\cdot 273-T_t)]}\int_y^1\frac{y^{4.5}}{y+\eta}\,dy,$$

where $\eta=\frac{7.6\ l^2T_t^2r}{10^5g\alpha[3T_t(273-T_t)-\alpha r(2\cdot 273-T_t)]}$,

$y=T_a/T_t$.

The pressure change at the surface due to convergence in the superior air is given by

$$(19)\qquad \frac{\partial p_o}{\partial t}=86400\cdot 0.98\left[\frac{\partial I_2}{\partial r}+\frac{I_2}{r}\right]mb/day.$$

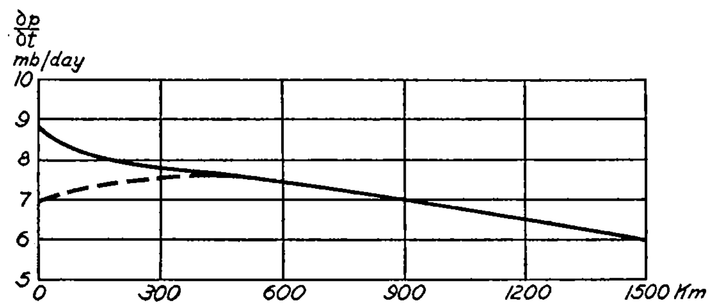

FIGURE 8.—Pressure tendency profile across the circular anticyclone.

In figure 8, the total pressure tendency profile, $\frac{\partial p_o}{\partial t}=\left(\frac{\partial p_o}{\partial t}\right)_1+\left(\frac{\partial p_o}{\partial t}\right)_2$, is plotted as a solid curve, and the profile due to convergence in the superior air, $\left(\frac{\partial p_o}{\partial t}\right)_2$, as the dashed curve, the latter being nearly coincident with the former from about 500 to 1,500 kilometers from the center since in this region the term $\left(\frac{\partial p_o}{\partial t}\right)_1$ is very small, Only near the center does convergence in the polar continental air cause an appreciable pressure rise which when added to the pressure rise due to convergence in the superior air shifts the maximum point of the rise from about 500 kilometers from the center to the center itself. However, if account is taken of anticyclogenesis, the inflow in the polar continental air will be greatly reduced and may even change sign. Hence, the sharp maximum at the center will be reduced and may even be shifted to a position slightly offset from the center.

We have just found that after 26 days of cooling the central increase of pressure in a polar anticyclone is about 8 mb/day. It can be shown that for a younger anticyclone the central increase of pressure is even larger. This increase of pressure, as we have seen, results from the convergence of air aloft into the cooled area. If this were the only process operative, after 26 days of cooling a very intense anticyclone would result having as a central pressure a value never observed. Evidently, there are other factors, of a dynamic nature, involved which prevent such a large accumulation of cold air.

The first of these factors is the increase in westerly winds resulting from the convergence of the superior air toward the center of cooling. The total absolute angular momentum of all the converging rings of air must be maintained and strong westerly winds will be created; the deflective effect of the earth's rotation acting on the rings will be larger than the prevailing pressure gradient, and so there will be a limit to the amount of convergence possible within a given high-level cyclone. To gage the magnitude of the westerly winds created by lateral displacement of rings of air, we may apply the following formula derived under the assumption that each individual ring of air preserves its absolute angular momentum:

$$u=l(r_0-r)$$

where r_0-r is the displacement of an individual ring, assumed small relative to r, the distance from the center; and $l=2\Omega\sin\phi$.

As found above, the convergence of the superior air is about 10 centimeters per second, and so the total displacement for 26 days is about 225 kilometers, creating a westerly wind of about 30 meters per second.

A second factor preventing the creation of large anticyclones is the release of the cold air at certain intervals in the form of an outburst of polar air such as is commonly observed in winter over large portions of the earth's surface. The release of the cold air does not seem to be a simple function of the intensity of the anticyclone or of the pressure gradient; nor is the cold air always galvanized into motion by a wave (young cyclone) moving along the front. Indeed, very often the cold air outbreaks seem to take place in North America without the aid of any cyclone, although usually, in this case, one forms to the left of the current after the outflow has begun. The factors favoring the release of cold air form a separate problem; but it seems as if one should look for a criterion involving the momentum and temperature structure of the westerly winds above the polar wedge, and the effects of lateral mixing.

THE SECOND APPROXIMATION FOR LINEAR FLOW

If the computations described above for linear flow are performed without neglecting variations in p_0, then equation (5) becomes

$$(20)\qquad G_t=\frac{gh_t\alpha}{lT_t}+G_o,$$

where G_0 is the geostrophic velocity at the surface, $\frac{1}{l\rho_o}\frac{\partial p_o}{\partial r}$, and G_t is the geostrophic velocity at the height h_t; the isallobaric velocity [equation (6)] becomes

$$(21)\qquad \frac{1}{l}\frac{\partial G_t}{\partial t}=\frac{gh_t\alpha\beta}{l^2T_t^2}+\frac{1}{l}\frac{\partial G_o}{\partial t};$$

the mass inflow in the polar continental air [equation (8)] becomes

$$(22)\qquad I_1=\frac{p_o}{gl}\left[\frac{R/m\ \alpha\beta}{lT_t}[1-(1+x)e^{-x}]+[1-e^{-x}]\frac{\partial G_o}{\partial t}\right]\text{gm/cm/sec},$$

where $x=\frac{H_t}{H}=4.5\frac{273-T_t}{T_t}$.

Likewise in the case of the superior air, equations (10), (11), and (13) become, respectively,

$$G_a=\frac{T_a}{T_i}\left[\frac{g\alpha H_i}{lT_i}+G_o\right], \tag{23}$$

$$\frac{1}{l}\frac{\partial G_a}{\partial t}=\frac{g\alpha\beta}{l^2}\frac{10^5}{7.6}\left(\frac{2{\cdot}273-T_i}{T_i^3}\right)T_a+\frac{T_a}{lT_i}\left(\frac{\partial G_o}{\partial t}+\frac{\beta}{T_i}G_o\right), \tag{24}$$

$$I_2=\frac{10^5 p_o e^{-\frac{H_i}{H}}}{7.6{\cdot}5.5\frac{R}{m}lT_i}\left[1-\left(\frac{T_a}{T_i}\right)^{5.5}\right]\left\{\frac{10^5 g\alpha\beta(2{\cdot}273-T_i)}{7.6\,lT_i}+T_i\frac{\partial G_o}{\partial t}+\beta G_o\right\}\text{gm/cm/sec.} \tag{25}$$

To find the surface pressure distribution at time t, it is necessary to solve the following equation for p_0:

$$\frac{\partial p_o}{\partial t}=0.98\times 86400\frac{\partial I}{\partial r}, \tag{14}$$

where $I=I_1+I_2$, and I_1, I_2 are given by equations (22) and (25).

As can be seen after writing G_0 in terms of p_0, and performing the required differentiations, a partial, nonlinear differential equation in p_0, with variable coefficients, results. The most practicable method of finding a solution appears to be by successive approximations. First, the pressure profile is computed when variations in p_0 (or G_0) are neglected; this can be easily done as shown above. Then the surface geostrophic wind distribution (G_0) is found; and also, by numerical methods, the surface isallobaric velocity distribution $\left(\frac{1}{l}\frac{\partial G_o}{\partial t}\right)$. These latter two quantities are then substituted in equations (22) and (25), resulting in a second approximation for I_1 and I_2. By numerical methods $\frac{\partial I_1}{\partial r}$ and $\frac{\partial I_2}{\partial r}$ are found, and therefore also the second approximation for $\frac{\partial p_o}{\partial t}$. The process may then be repeated to give further approximations. However, because of the limitations in the numerical methods used, only the second approximation for the pressure tendency profile could be found with any assurance; this is shown in figure 7 as the dashed curve. As should be expected, the second approximation shows a smaller increase of pressure; the difference is especially marked at the center of the anticyclone, where the second approximation is about 24 percent smaller than the first, while at the periphery it is only 14 percent smaller. It has not yet been shown mathematically whether the successive approximations converge and, if so, whether the convergence is so rapid that the second solution is sufficiently accurate. It does not seem worth while to spend more time on the mathematical problems involved, since, as pointed out above, the treatment here has neglected certain important dynamical considerations, such as conservation of angular momentum; this omission the author hopes to correct in a later paper.

CONSTANTS OF THE MODEL

26 days of cooling at a rate of 1.35° C./day;
Vertical growth of the polar continental air ~180 meters/day;
Horizontal growth of the polar continental air ~58 km/day;
Slope of the front ~1/300;
Inflow velocity of the superior air ~9 km/day;
Vertical ascent of the superior air ~30 meters/day;
Sinking of the front by contraction of the cooled air ~10 to 20 meters/day.

The superior air undergoes very little vertical displacement as it moves toward the center, since the ascent caused by motion up the frontal surface is almost compensated by the vertical contraction of the lower cooled air. At the same time, of course, the superior air is being transformed into polar continental air at a rate of 180 meters/day vertically; and, likewise, the discontinuity in inflow velocity is raised vertically at the same rate. The comparatively large inward velocities which existed in the superior air before it was transformed into polar continental air will no longer be maintained and will disappear by mixing with the slower moving air.

SUMMARY

When cooling of air occurs over a certain region, the air contracts, the isobaric surfaces are lowered, and a compensating inflow of air aloft raises the surface pressure and gives rise to a surface anticyclone. An explanation of the mechanics of the compensating inflow has been attempted on the basis of the Brunt-Douglas isallobaric velocity component, which is directed into the deepening cyclone aloft (polar cyclone). The vertical distribution of this inflow is studied; and it is found that in going through the front from the polar continental air to the air above, a many-fold increase in isallobaric velocity occurs, showing that almost all the increase in surface pressure results from convergence in the air above the lower cooled air.

At any given time in the life history of the growing polar anticyclone, it is possible to construct surface pressure tendency profiles, and the magnitude of the increases seems to be in satisfactory agreement with those observed on weather maps.

The next step in the problem is to include certain dynamical reasoning omitted in this preliminary treatment and then to explain the release of these large masses of cold air, which occurs in a discontinuous manner, sometimes with no apparent clue in the shapes of surface isobars or in 3- and 12-hour pressure changes.

METEOROLOGICAL ASPECTS OF HAILSTORMS IN NEBRASKA

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This paper presents the results obtained from an intensive study of the available data on hailstorms in Nebraska, which cover the 13-year period 1924–36, inclusive. These data from the files of the United States Weather Bureau office at Lincoln are the result of observations made by voluntary weather observers at cooperative Weather Bureau stations located in various parts of the State. The reports contain data pertaining to location, width, length, and direction of movement of the individual hailstorms. Each report was carefully checked by the officials of the Weather Bureau and thus obvious errors and superficial estimates were corrected. Few storms are recorded earlier than April or later than September. Even though hail should fall during the late fall and winter months a relatively small amount of damage is done to crops. Hence, only the months April to September, inclusive, are here considered.

Hail occurs only during the passage of a thunderstorm; and records indicate that destructive hail occurs in only a comparatively small number of thunderstorms. Of the